FULLY DECENTRALIZED JOINT LEARNING OF PERSONALIZED MODELS AND COLLABORATION GRAPHS

Aurélien Bellet (Inria MAGNET)

Includes work with:
M. Tommasi, P. Vanhaesebrouck (Inria, Univ. Lille)
R. Guerraoui, M. Taziki (EPFL)
V. Zantedeschi (Univ. St-Etienne)

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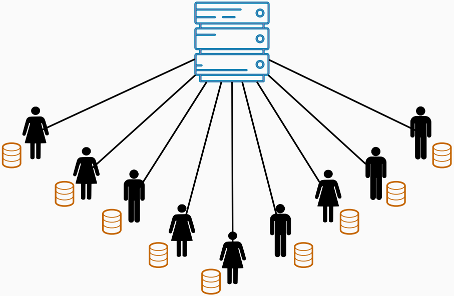
CONTEXT AND MOTIVATION

LEARNING ON CONNECTED DEVICES DATA

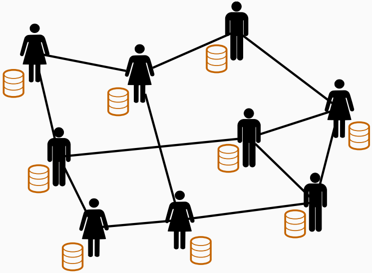
- Connected devices are widespread and collect increasingly personal data
- Ex: browsing logs, health, speech, accelerometer, geolocation
- Great opportunity to provide personalized services
- Two classic strategies:
    - Centralize data from all devices: limited user control, privacy and security issues, communication/infrastructure costs
    - Learn on each device separately: poor utility for many users
- **Goal**: find a sweet spot between these two extremes

RELATED WORK: FEDERATED LEARNING



- Coordinator-clients architecture [McMahan et al., 2017]
- Iterates over the following (synchronous) steps:
  - Clients send model updates computed on local data
  - Coordinator aggregates and sends the new model back to clients
- Heavy dependence on coordinator: single point of failure and scalability issues with large number of clients

RELATED WORK: FULLY DECENTRALIZED LEARNING



- Peer-to-peer and asynchronous exchanges along sparse communication graph
- No single point of failure as in classic federated learning
- Scalability-by-design to many devices (see e.g., [Lian et al., 2017])

OUR APPROACH: DESIRED PROPERTIES

1. Keep data on the device of the users
2. Learn personalized models in collaborative fashion
3. Learn and leverage a graph of similarities between users
4. Decentralized algorithms to scale to large number of devices

And also (not in this talk):

5. Differential privacy guarantees [Bellet et al., 2018]
6. Low-communication via greedy boosting [Zantedeschi et al., 2019]

PROBLEM FORMULATION

- A set $[\![n]\!] = \{1, \dots, n\}$ of users (or agents)
- Each user has a personal distribution over common feature space $\mathcal{X}$ and label space $\mathcal{Y}$ $\rightarrow$ personal supervised learning task
- User i has local dataset $\mathcal{S}_i = \{(x_i^j, y_i^j)\}_{j=1}^{m_i}$ of size $m_i \geq 0$ drawn from personal distribution
- His/her goal is to learn a model $\theta_i \in \mathbb{R}^p$ which generalizes well to new examples from personal distribution

- Let $\ell : \mathbb{R}^p \times \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$ be a loss function
- In isolation, user i can learn a purely local model by ERM

$$\theta_i^{loc} \in \underset{\theta \in \mathbb{R}^p}{\arg\min}\, \mathcal{L}_i(\theta; \mathcal{S}_i) = \frac{1}{m_i} \sum_{j=1}^{m_i} \ell(\theta; x_i^j, y_i^j) + \lambda_i \|\theta\|^2, \text{ with } \lambda_i \geq 0$$

- Poor generalization when local data is scarce
- **Goal:** improve upon θ_i^{loc} by discovering relationships between personal tasks and leveraging them to learn better personalized models in a fully decentralized setting

DECENTRALIZED SETTING

- **Asynchronous time model**: each user has a local Poisson clock and wakes up when it ticks [Boyd et al., 2006]
- Equivalently: single clock (with counter t, unknown to the users) ticking when one of the local clocks ticks
- **Communication model**: semantic overlay over complete graph to restrict communication to pairs of most similar users
- We call this overlay the collaboration graph: undirected, weighted graph $\mathcal{G}_w = ([n], w)$ with edge weight $w_{ij} \geq 0$ reflecting similarity between the learning tasks of users i and j
- **Our method:** learn sparse collaboration graph and personalized models by optimization of a joint objective

JOINT OPTIMIZATION PROBLEM

- Learn personalized models $\Theta \in \mathbb{R}^{n\times p}$ and graph weights $w \in \mathbb{R}_{\geq 0}^{n(n-1)/2}$ as solutions to [Zantedeschi et al., 2019]:

$$\min_{\substack{\Theta\in\mathbb{R}^{n\times p}\\ w\in\mathbb{R}_{\geq 0}^{n(n-1)/2}}} J(\Theta, w) = \sum_{i=1}^{n} d_i c_i \mathcal{L}_i(\theta_i; \mathcal{S}_i) + \frac{\mu}{2}\sum_{i<j} w_{ij}\|\theta_i - \theta_j\|^2 + \lambda g(w),$$

- $c_i \in (0,1] \propto m_i$: confidence of user i, $d_i = \sum_{j\neq i} w_{ij}$: degree of i
- Trade-off between having accurate models on local dataset and smoothing models along the graph
- Term $g(w)$: avoid trivial graphs, encourage desirable properties
- Flexible relationships: μ interpolates between learning purely local models and a shared model per connected component

OUTLINE OF THE PROPOSED APPROACH

- Problem not jointly convex in Θ and w, but is typically bi-convex
- Natural approach: alternating optimization over Θ and w
- I will present a decentralized algorithm to learn the models given the graph (communication along edges of the graph)
- I will then present a decentralized algorithm to learn a (sparse) graph given the models (communication through peer sampling)

LEARNING MODELS GIVEN THE GRAPH

PROPERTIES OF OBJECTIVE FUNCTION

- For fixed graph weights, denote $f(\Theta) := J(\Theta, w)$
- Assume local loss $\mathcal{L}_i$ has L_i^{loc}-Lipschitz continuous gradient
- Then $\nabla_\Theta f$ is L_i-Lipschitz w.r.t. block Θ_i with $L_i = d_i(\mu + c_i L_i^{loc})$
- Can also assume that $\mathcal{L}_i$ is σ_i^{loc}-strongly convex where $\sigma_i^{loc} > 0$
- Then f is σ-strongly convex with $\sigma \geq \min_{1 \leq i \leq n}[d_i c_i \sigma_i^{loc}] > 0$

- Denote neighborhood of user i by $N_i = \{j : w_{ij} > 0\}$
- Initialize models $\Theta_i(0) \in \mathbb{R}^{n \times p}$
- At step $t \geq 0$, a random user i wakes up:
  1. user i updates its model based on information from neighbors:

$$\Theta_i(t+1) = \Theta_i(t) - \frac{1}{\mu + c_i L_i^{loc}} \left(c_i \nabla \mathcal{L}_i(\Theta_i(t); \mathcal{S}_i) - \mu \sum_{j \in N_i} \frac{w_{ij}}{d_i} \Theta_j(t) \right)$$

  2. user i sends its updated model $\Theta_i(t+1)$ to its neighborhood N_i
- The update is a trade-off between a local gradient step and a weighted average of neighbors' models

CONVERGENCE RATE

Proposition ([Bellet et al., 2018])

For any $T > 0$, let $(\Theta(t))_{t=1}^{T}$ be the sequence of iterates generated by the algorithm running for T iterations from an initial point $\Theta(0)$. When f is σ-strongly convex in Θ, we have:

$$\mathbb{E}\left[f(\Theta(T)) - f^\star\right] \leq \left(1 - \frac{\sigma}{nL_{max}}\right)^T \left(f(\Theta(0)) - f^\star\right),$$

where $L_{max} = \max_i L_i$.

- Essentially follows from coordinate descent analysis [Wright, 2015]
- Can obtain convergence in $O(1/T)$ in convex case
- Can extend analysis to the case where random noise is added to ensure differential privacy [Bellet et al., 2018]

LEARNING THE GRAPH GIVEN MODELS

$$\min_{\substack{\Theta \in \mathbb{R}^{n \times p} \\ w \in \mathbb{R}_{\geq 0}^{n(n-1)/2}}} J(\Theta, w) = \sum_{i=1}^{n} d_i c_i \mathcal{L}_i(\theta_i; \mathcal{S}_i) + \frac{\mu}{2} \sum_{i<j} w_{ij} \|\theta_i - \theta_j\|^2 + \lambda g(w),$$

- Our algorithm can deal with weight and degree-separable g
- Inspired by [Kalofolias, 2016], we can set $\lambda = \mu$ and define

$$g(w) = \beta \|w\|^2 - 1^T \log(d + \delta) \quad \text{(with } \delta \text{ small constant)}$$

- Log barrier on the degree vector d to avoid isolated users and L_2 penalty on weights to control the graph sparsity
- Denoting $h(w) := J(\Theta, w)$ for fixed Θ, then h is strongly convex

DECENTRALIZED ALGORITHM

- We rely on decentralized peer sampling [Jelasity et al., 2007] to allow users to communicate with a set of κ random peers
- Initialize weights $w(0)$, set parameter $\kappa \in [\![n-1]\!]$
- At each step $t \geq 0$, a random user i wakes up:
  1. Draw a set $\mathcal{K}$ of κ users and request their model, loss and degree
  2. Update the associated weights $w(t+1)_{i,\mathcal{K}} = (w(t+1)_{ij})_{j\in\mathcal{K}} \in \mathbb{R}^\kappa$:

     $$w(t+1)_{i,\mathcal{K}} \leftarrow \max\left(0, w(t)_{i,\mathcal{K}} - \frac{1}{L_\kappa}[\nabla h(w(t))]_{i,\mathcal{K}}\right)$$

     where $L_\kappa = 2\mu(\frac{\kappa+1}{\delta^2} + \beta)$ is the block Lipschitz constant of $\nabla h(w)$
  3. Send each updated weight $w(t+1)_{ij}$ to the associated user $j \in \mathcal{K}$

Theorem ([Zantedeschi et al., 2019])

For any $T > 0$, let $(w(t))_{t=1}^{T}$ be the sequence of iterates generated by the algorithm running for T iterations from an initial point $w(0)$. We have $\mathbb{E}[h(w^{(T)}) - h^*] \leq \rho^T(h(w^{(0)}) - h^*)$ where ρ is given by

$$\rho = 1 - \frac{4}{n(n-1)} \frac{\kappa\beta\delta^2}{\kappa + 1 + 2\beta\delta^2}$$

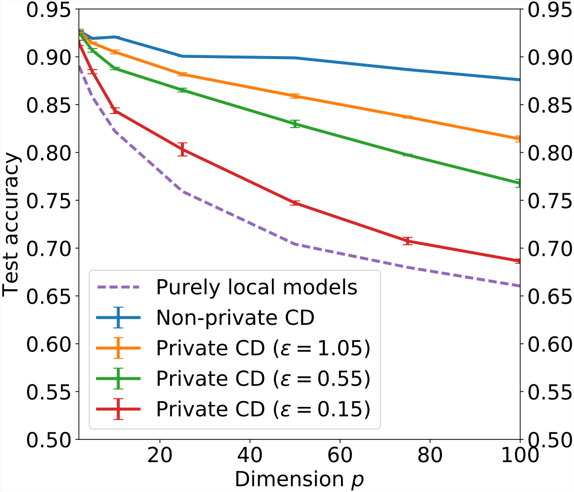
- Can be seen as an instance of proximal coordinate descent with an overlapping block structure
- κ can be used to trade-off between communication cost and convergence speed (more on this in [Zantedeschi et al., 2019])

NUMERICAL EXPERIMENTS

EXPERIMENTS: COLLABORATIVE LINEAR CLASSIFICATION

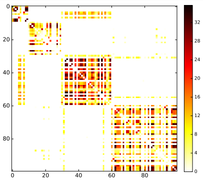
- We consider a set of $n = 100$ users and a synthetic linear classification task in $\mathbb{R}^p$ (we use the hinge loss)
- Each user is associated with an (unknown) target linear model
- Each user i receives a random number m_i of samples with label given by the prediction of target model (plus noise)
- We can build an “oracle” collaboration graph based on the angle between target models (note: this is cheating!)

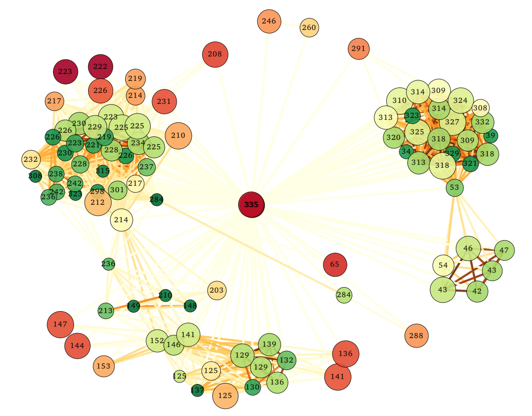
- Results when using the oracle graph



EXPERIMENTS: COLLABORATIVE LINEAR CLASSIFICATION

- We show that the learned topology adapts to the problem, unlike classic heuristics (e.g., k-NN graph)
- Below we approximately recover the cluster structure, and prediction accuracy is close to that of the oracle graph





- Real datasets that are naturally collected at the user level
- Number of users n from 23 to 190, no task similarity available
- Linear models, and nonlinear ensembles [Zantedeschi et al., 2019]
- Our approach clearly outperforms global and local models
- Greedily trained nonlinear ensembles achieve better accuracy under communication budget (see [Zantedeschi et al., 2019])

Dataset	Global-lin	Local-lin	**Ours-lin**	Global-nonlin	Local-nonlin	**Ours-nonlin**
Harws	93.64	92.69	**96.31**	94.34	93.16	95.70
Vehicle	87.11	90.38	**91.37**	88.02	90.59	90.81
Computer	62.18	60.68	69.08	69.16	66.61	**72.09**
School	57.06	70.43	71.92	69.16	66.61	**72.22**

bold blue = best, regular blue = second best

FUTURE WORK

SOME FUTURE WORK

- Extend analysis to nonconvex setting (deep neural nets)
- Use the graph to smooth predictions rather than model parameters
- Learn graph weights as statistical estimates of some distance between data distributions and prove generalization guarantees
- Dynamic setting: data arrives sequentially, users join/leave
- Robustness to malicious parties [Dellenbach et al., 2018]

Thank you for your attention!

Questions?

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PRIVACY ISSUES WHEN LEARNING THE MODELS

- In some applications, data may be sensitive and users may not want to reveal it to anyone else
- In the previous algorithm, users never communicate their local data but exchange sequences of models computed from data
- Consider an adversary observing all the information sent over the network (but not the internal memory of users)
- **Goal:** formally guarantee that no/little information about the local dataset is leaked by the algorithm

(ϵ, δ)-Differential Privacy [Dwork, 2006]

Let $\mathcal{M}$ be a randomized mechanism taking a dataset as input, and let $\epsilon > 0, \delta \geq 0$. We say that $\mathcal{M}$ is (ϵ, δ)-differentially private if for all datasets $\mathcal{S}, \mathcal{S}'$ differing in a single data point and for all sets of possible outputs $\mathcal{O} \subseteq \text{range}(\mathcal{M})$, we have:

$$Pr(\mathcal{M}(\mathcal{S}) \in \mathcal{O}) \leq e^{\epsilon} Pr(\mathcal{M}(\mathcal{S}') \in \mathcal{O}) + \delta.$$

- Guarantees that the output of $\mathcal{M}$ is almost the same regardless of whether a particular data point was used
- Robust to background knowledge that adversary may have
- Information-theoretic (no computational assumptions)
- Composition property: the combined output of two (ϵ, δ)-DP mechanisms (run on the same dataset) is $(2\epsilon, 2\delta)$-DP

1. Replace the update of the algorithm by

$$\widetilde{\Theta}_i(t+1) = \widetilde{\Theta}_i(t) - \frac{1}{\mu + c_i L_i^{loc}} \Big(c_i (\nabla \mathcal{L}_i(\widetilde{\Theta}_i(t); \mathcal{S}_i) + \eta_i) - \mu \sum_{j \in N_i} \frac{W_{ij}}{d_i} \widetilde{\Theta}_j(t) \Big),$$

   where $\eta_i \sim Laplace(0, s_i)^p \in \mathbb{R}^p$
2. user i then broadcasts noisy iterate $\widetilde{\Theta}_i(t+1)$ to its neighbors

- In our setting, the output of our algorithm is the sequence of users' models sent over the network

Theorem ([Bellet et al., 2018])

Assume user i wakes up T_i times and use noise scale $s_i = \frac{L_0}{\epsilon_i m_i}$. Then for any initial point $\widetilde{\Theta}(0)$ independent of $\mathcal{S}_i$, the algorithm is $(\bar{\epsilon}_i, 0)$-DP with $\bar{\epsilon}_i = T_i \epsilon_i$.

PRIVACY/UTILITY TRADE-OFF

Theorem ([Bellet et al., 2018])

For any $T > 0$, let $(\widetilde{\Theta}(t))_{t=1}^{T}$ be the sequence of iterates generated by T iterations. We have:

$$\mathbb{E}\left[(\widetilde{\Theta}(T)) - ^\star\right] \leq \rho^T \left((\widetilde{\Theta}(0)) - ^\star\right) + \left(\frac{1}{(1-\rho)Cn}\sum_{i=1}^{n}\left(d_i c_i s_i\right)^2\right)\left(1-\rho^T\right)$$

- Second term gives additive error due to noise
- Sweet spot: the less data, the more noise added by the user, but the least influence in the network
- T rules a trade-off between optimization error and noise error