DeepSphere: towards an equivariant graph-based spherical CNN

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We have spherical data. How can we use a neural network with them?

- Intrinsically spherical data:
  - cosmic microwave background
  - daily temperature
  - brain activity (MEG)

DeepSphere:
1. Model the sampled sphere as a graph.
2. Use a Laplacian-based graph neural network.

=> Efficient and equivariant spherical CNN.

Why is your graph convolution spherical and equivariant?

Observation: the graph Laplacian’s eigenvectors are close to the spherical harmonics.

Reasoning:
1. The graph Fourier transform is similar to the spherical harmonic transform.
2. Convolution is a multiplication in the spectral domain.
3. The graph convolution is close to the spherical convolution.

Consequence: graph convolution is (almost) rotation equivariant.

Spatial properties of graph filters:
- Invariant to localization => equivalence to SO(3) rotations
- Isotropic kernel

Then, how is it different from spherical convolution? (used in [Cohen] and [Esteves])

- Equivariant to rotations (almost).
- Fast: $O(N)$ vs $O(N^{3/2})$.
- Flexible: accommodates any sampling and partial observations.
- Easy to implement (use general & efficient graph NN implementations).
- Invariant instead of equivariant to the 3rd rotation (isotropic filters).

Graph NNs only do same-equivariance and invariance.

Recognition of 3D shapes (SHREC-17):
- Same accuracy as [Cohen] and [Esteves].
- Computationally much more efficient.
- Less parameters.

=> Equivalence to 3rd rotation is an unnecessary price to pay.

Task: Discriminate against cosmological models.
The goal is to identify the model that best fits our observations of the universe.

Result: DeepSphere beats ConvNet on 2D projections and SVM baselines.
Too many pixels (12M) for [Cohen] and [Esteves] (which were tested on 10k pixels).

DeepSphere v2 (coming soon):
- Empirical correspondence of the eigenspaces.
- Proof of convergence.

Different graphs lead to different symmetries.
- Geometric graphs: translations and rotations.
- General graphs: node permutation.

You pay for what you use on irregular samplings, but equivariance needs investigation.

References
- Cohen, Geiger, Köhler, Welling, Spherical CNNs, 2018.
- Esteves, Allen-Blanchette, Makadia, Daniilidis, Learning SO(3) equivariant representations with spherical CNNs, 2018.

Github
https://github.com/SwissDataScienceCenter/DeepSphere